Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: analysis and approaches
## Standard level
## Paper 1

15 May 2025

**Zone A** afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

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1 hour 30 minutes

### Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

**1.** [Maximum mark: 5]

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$ [1]

(b) $3 \ln 2$ [2]

(c) $-\ln \frac{1}{2}$ [2]

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2. [Maximum mark: 5]

Consider the function $f(x)=\frac{4x^3}{3}-16x$, where $x \in \mathbb{R}$.

The graph of $y=f(x)$ has a local minimum point at $(p, q)$ where $p>0$.

Find the value of $p$ and the value of $q$.

**3.** [Maximum mark: 7]

Bob invests $1000$ dinar in an account which pays a nominal annual interest rate of $4\%$ compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1+k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of $k$. [1]

(b) Expand and simplify $(1+x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

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**4.** [Maximum mark: 4]

Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$.

**5.** [Maximum mark: 6]

Consider events $A$ and $B$ such that $P(A') = P(A \cup B) = \frac{3}{4}$ and $P(B|A) = \frac{2}{3}$.

(a) Find $P(A \cap B)$. [3]

(b) Show that events $A$ and $B$ are independent. [3]

6. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \ldots, F_9, F_{10}$.

Picture frame $F_1$ has width $4\,\text{cm}$ and height $5\,\text{cm}$.

The width and height of picture frame $F_n$, are each increased by $50\%$ to generate the width and height of the next picture frame $F_{n+1}$, for $n \in \mathbb{Z}^+$, $1 \le n \le 9$.

(a) (i) Show that the area of picture frame $F_n$ is $20\left(\frac{9}{4}\right)^{n-1}\text{cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^a - 1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^4\text{cm}^2$, where $q \in \mathbb{Q}^+$. [3]

Do **not** write solutions on this page

## Section B

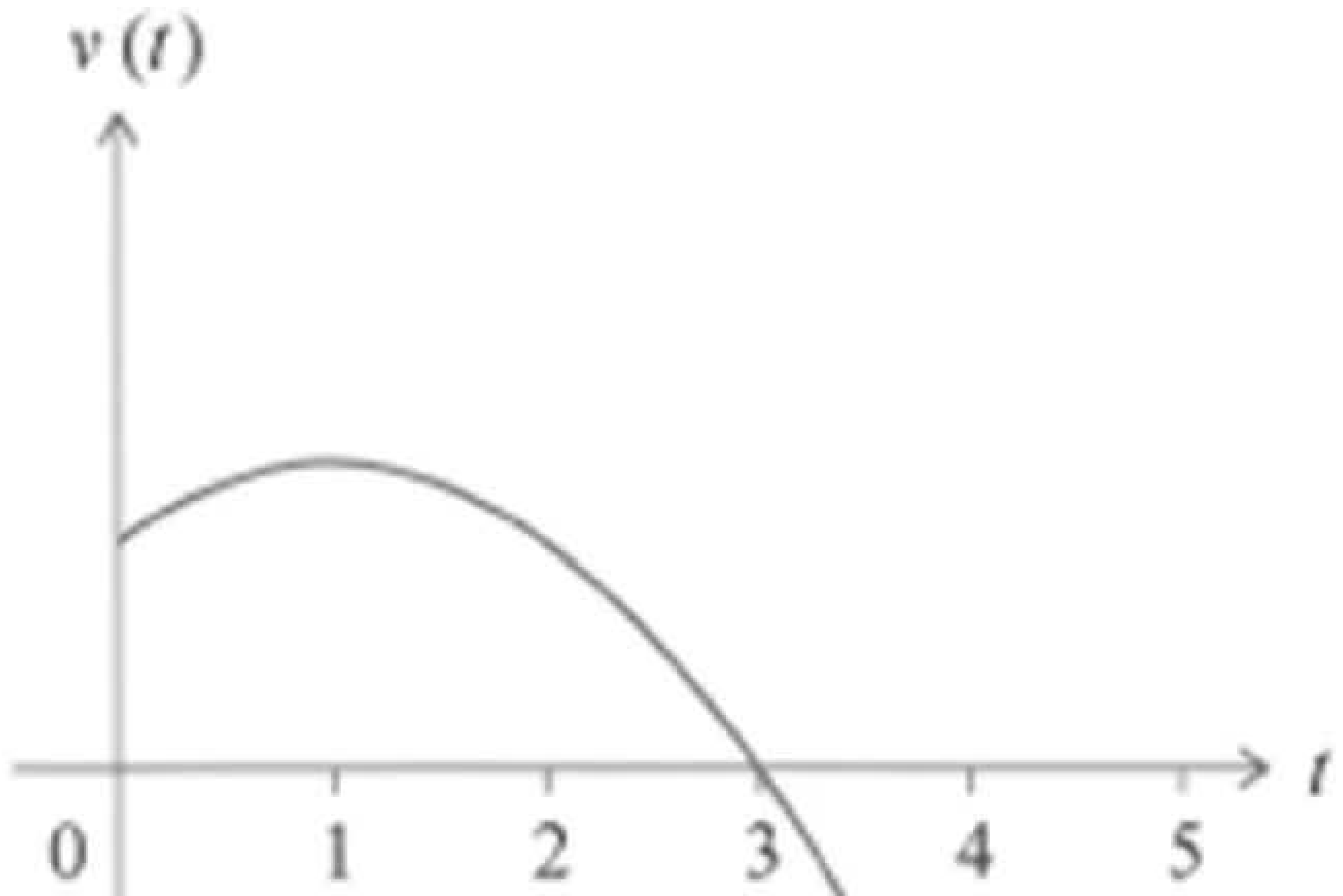
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

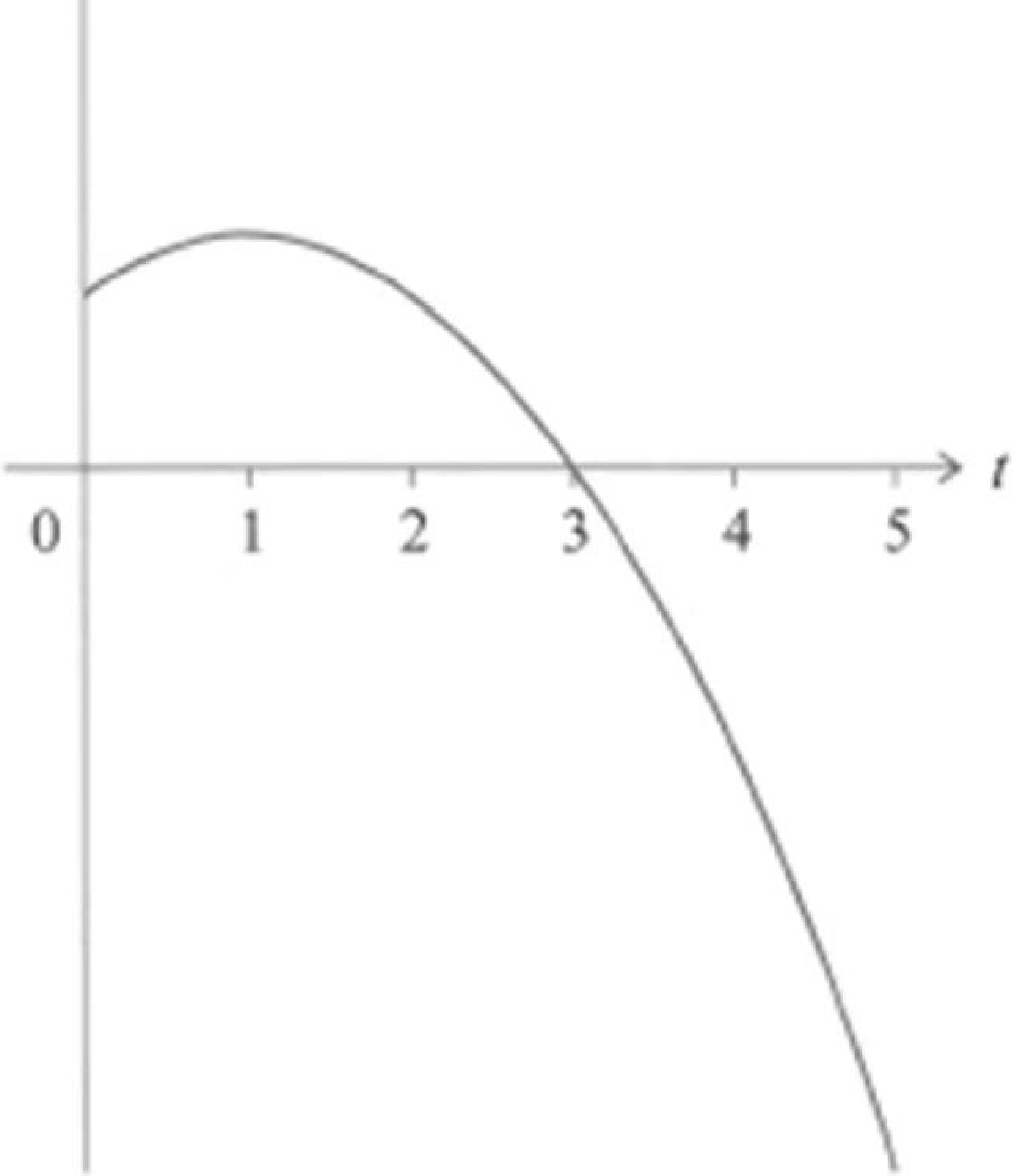
**7.** [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v\,\mathrm{m\,s^{-1}}$, at time $t$ seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \le t \le 5$.

The graph of $v$ is shown in the following diagram.






The graph of $v$ has a local maximum point where $t = 1$ and intersects the $t$-axis at $t = 3$.

(a) Determine the object's

(i) maximum velocity;

The graph of $v$ has a local maximum point where $t = 1$ and intersects the $t$-axis at $t = 3$.

(a) Determine the object's

(i) maximum velocity;

(ii) maximum speed. [4]

At $t = T$, the object changes direction.

(b) (i) Write down the value of $T$.

(ii) Find the distance travelled by the object in the first $T$ seconds. [5]

(c) Determine whether the object returns to its initial position during the time period $0 \leq t \leq 5$, justifying your answer. [4]

Do **not** write solutions on this page

**8.** [Maximum mark: 15]

The function $f$ is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \leq 40$. [4]

The function $g$ is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]

**9.** [Maximum mark: 17]

A solid cylinder has height $h$ cm and base radius $R$ cm.

The cylinder fits exactly inside a hollow sphere of radius $r$ cm.

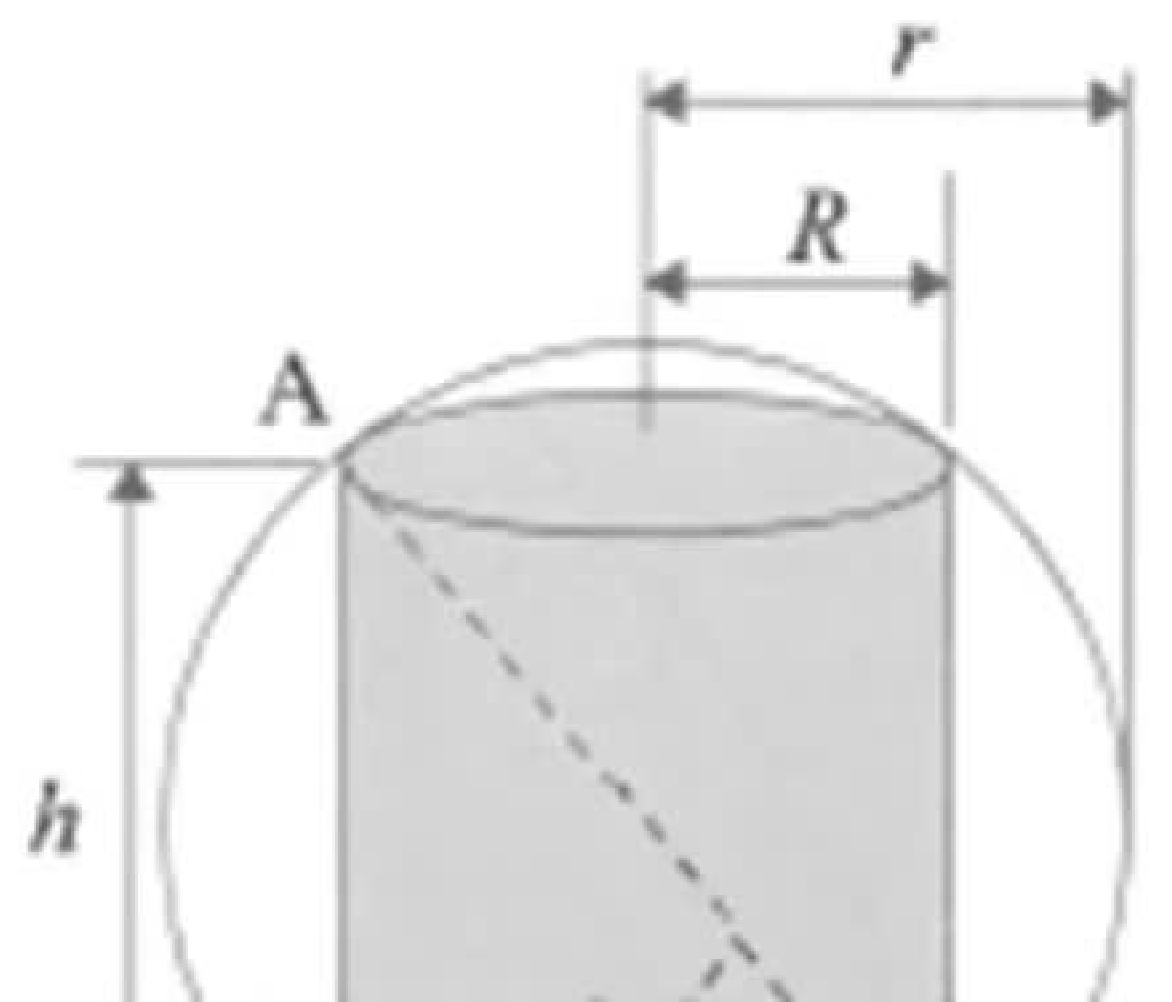
Points A, B and C are points where the surface of the cylinder touches the surface of the sphere.

The line segment [AB] is a diameter of the sphere.

The line segment [BC] is a diameter of the base of the cylinder and $\mathrm{A\hat{B}C} = \theta$.

This information is shown on the following diagram.

**diagram not to scale**



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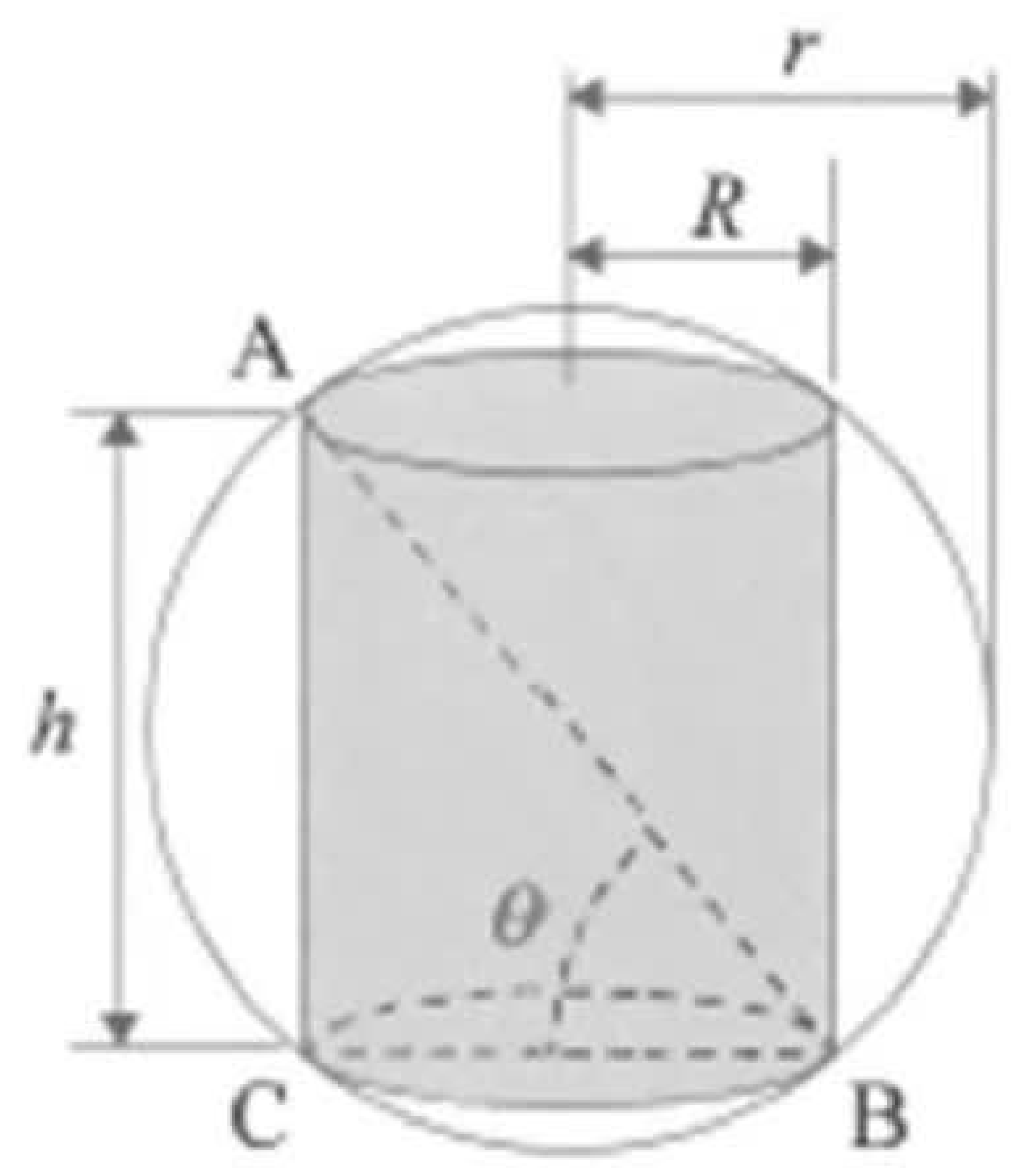

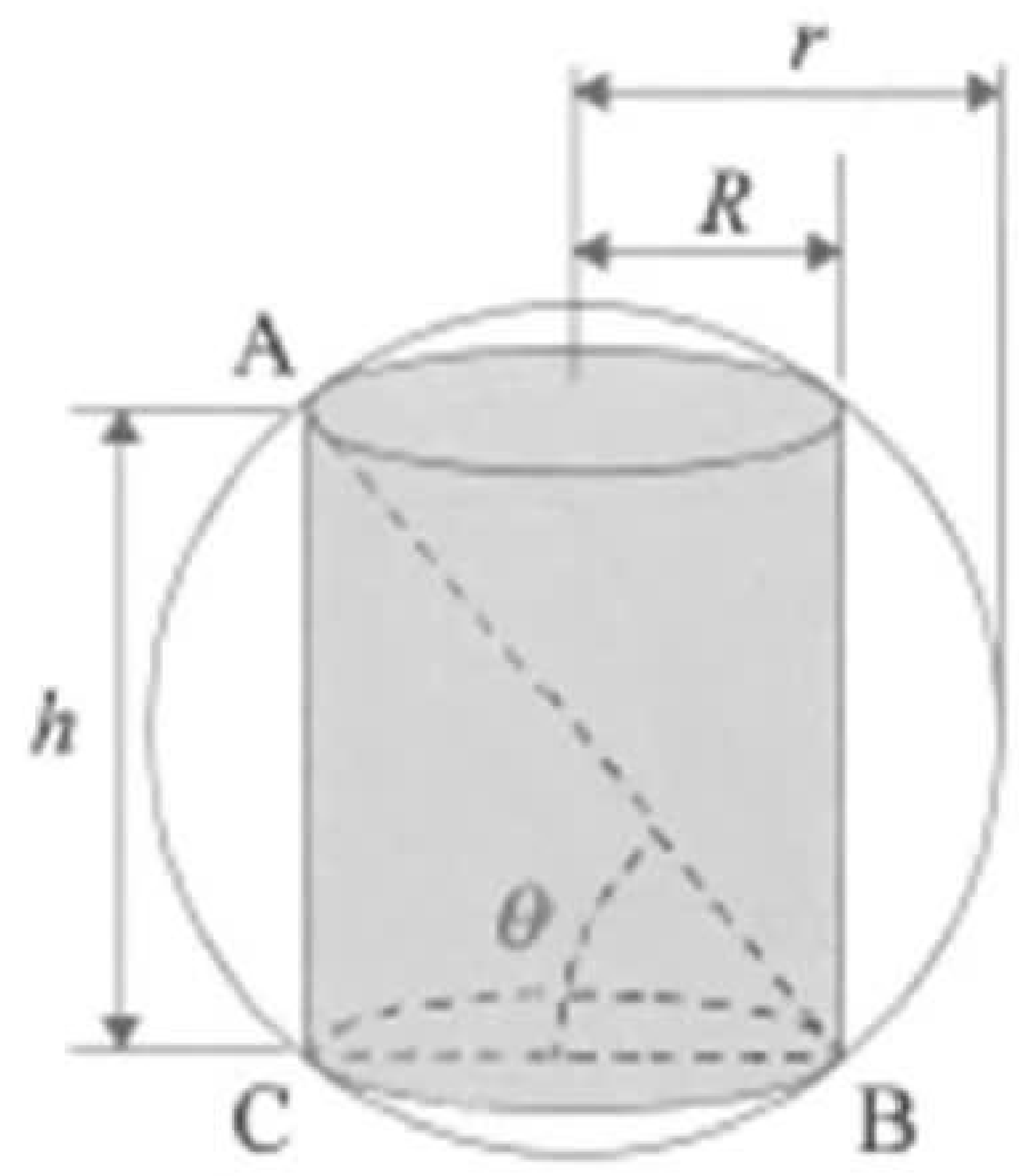


(a) (i) By considering triangle ABC, show that $R = r\cos\theta$.

(ii) Find an expression for $h$ in terms of $r$ and $\theta$. [4]

(b) Hence or otherwise, show that the total surface area, $S\,\text{cm}^2$, of the cylinder is given by $S = 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$ . [4]

The external surface area of the sphere is $2S$.

(c) Show that $\tan\theta = 2$. [4]

The volume of the cylinder is $V\,\text{cm}^3$.

(d) Find $V$, giving your answer in the form $p\pi r^3\sqrt{5}$, where $p \in \mathbb{Q}^+$. [5]